

# Spectrum of Strange Baryon Resonances in Hypercentral Constituent Quark Model

DHIRENDRA SINGH

Physics Department,  
D.B.S. (P.G.) College, Kanpur, INDIA.

(Received on: December 4, 2012)

## ABSTRACT

The mass spectra of strange baryons resonances calculated within the framework of a hypercentral constituent quark model. The confinement potential is assumed as a combination of coulomb like term, a linear confining term plus a harmonic oscillator potential. We study the symmetries of baryon spectrum using a very simple approach based on Gursey - Redicati mass formula. The results of mass spectra of strange baryons resonances is found to be in excellent agreement with the experimental values reported by particle data group over other theoretical model predictions.

**Keywords:** Hypercentral constituent quark models, Phenomenological quark model, Potential models, Flavour symmetries in particles and fields.

## INTRODUCTION

During last few years a significant experimental progress has been achieved in studying the baryon properties. Different versions of constituent quark models<sup>1-5</sup> have been proposed for the description of baryons in the last decades. These models have in common fact that the three quark interaction can be separated by two parts: The first one containing the confinement interaction is spin and flavour independent and it is therefore SU(6) invariant, while the second violates the SU(6) symmetry.

In this paper we study the symmetries of baryon spectrum using a very simple approach based on Gursey - Redicati mass formula<sup>6</sup>. The model proposed is a simple constituent quark model, where the SU(6) invariant part of the Hamiltonian is the same as hypercentral constituent quark model<sup>7</sup> and where the SU(6) symmetry is broken by Gursey- Redicati inspired interaction.

## Theoretical Model

The model Hamiltonian for the strange baryon is expressed in terms of the

Jacobin coordinates  $(\rho, \lambda)$  as well as hypercentral coordinates are (r) as be written as

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(r) \quad (1)$$

For the complete description of strange baryons, we have considered an interaction potential of the form

$$V(r) = K_0 + V_{h.o.}(r) + V_{conf.}(r) - V_{hyc.}(r) \quad (2)$$

Where  $V_{h.o.}(r)$  is a added six dimensions harmonic oscillator potential, which has a two body character, and turns out to be exactly hypercentral.

$$V_{h.o.}(r) = \sum_{i < j}^{i=3} \frac{1}{2} k(r_i - r_j)^2 = \frac{3}{2} kx^2 = \eta x^2 \quad (3)$$

$V_{conf.}(r)$  is hyper- linear term gives rise quark confinement<sup>5</sup> due to large separations.

$$V_{conf.}(r) = kr \quad (4)$$

$V_{hyc.}(r)$  is a six dimensional hyper coulomb potential<sup>8</sup> which is attractive for small separations.

$$V_{hyc.}(r) = -\tau r^{-1} \quad (5)$$

where  $K_0, k, \eta$  and  $\tau$  are constants. the quark potential V, is supposed to depend on hyper radius r only, that is to be hypercentral. Therefore  $V=V(r)$  is in general a three body potential, since hyper radius r

depends on the coordinates of all three quarks. The following analytical solution of potential is employed<sup>9</sup>. The energy eigen values for the mode  $\nu = 0$  and grand angular momentum  $\gamma$  are given as follows.

$$E_{0\gamma} = (2\gamma + 6) \frac{\omega}{2} - \frac{2mr^2}{(2\gamma + 5)^2} \quad (6)$$

and the ground state normalized eigenfunctions are given as

$$\Psi_{0\gamma} = N_\gamma x^{-\frac{5}{2}} \phi_{0\gamma} = N_\gamma x^\gamma \exp \left( -\frac{m\omega}{2} x^2 - \frac{2mr}{(2\gamma + 5)} x^2 \right) \quad (7)$$

Similarly we can continue for other modes ( $\nu = 1, 2, 3, \dots$ ).

The hypercentral constituent quark model is fairly good for non strange baryon spectrum but in some cases splitting within the SU(6) baryon multiplets is provided by Gursey - Redicati mass formula<sup>10</sup>

$$M = M_0 + CS(S+1) + DY + E[T(T+1) - \frac{1}{4}Y^2] \quad (8)$$

where  $M_0$  is the average energy value of the SU(6) multiplets, S is the total spin, Y is the hypercharge and T is the total isospin of baryons. Eq.(8) can be written in terms of Casimir operators in the following way

$$M = M_0 + CC_2[SU_s(2) + DC_1[U_Y(1)] + E[C_2[SU_I(2) - \frac{1}{4}(C_1[U_Y(1)]^2)] \quad (9)$$

where  $C_2[SU_s(2)]$  and  $C_2[SU_I(2)]$  are the SU(2) (quadratic) Casimir operators for spin and isospin, respectively, and  $C_1[U_Y(1)]$  is

the Casimir for U(1) subgroup generated by the hypercharge Y.

Giannini *et al.* considered a dynamical spin-flavor symmetry  $SU_{SF}(6)$ <sup>11</sup> and describe the symmetry breaking mechanism by generalizing Eq.(9) as

$$M = M_0 + AC_2[SU_{SF}(6) + BC_2[SU_F(3)] + CC_2[SU_S(2)] + DC_1[U_Y(1)] + E[C_2[SU_I(2)] - \frac{1}{4}(C_1[U_Y(1)])^2] \quad (10)$$

It is a conditional generalized G R mass formula. Therefore, the strange baryon masses are obtained by three quark masses and the eigen energies( $E_{\nu\gamma}$ ) of the radial Schrödinger equation with the expectation values of  $H_{GR}$  as

$$M = \sum m_q + E_{\nu\gamma} + \langle H_{GR} \rangle \quad (11)$$

where  $\sum m_q$  are the constituent quark masses. for calculation the constituent quark masses are assumed ( $m_u = m_d = m$ ). In eqn. (11),  $H_{GR}$  is in the following form.

$$H_{GR} = AC_2[SU_{SF}(6) + BC_2[SU_F(3)] + CC_2[SU_S(2)] + DC_1[U_Y(1)] + E[C_2[SU_I(2)] - \frac{1}{4}(C_1[U_Y(1)])^2] \quad (12)$$

The expectation values of  $H_{GR}$ , ( $\langle H_{GR} \rangle$ ), is completely identified by the expectation values of Casimir operators<sup>12</sup>. For calculating the baryon masses, we choose a limited no. of well known strange parameters<sup>13</sup>.

**Table 1. Parameters in the model**

Parameter	Value
A	-7.92 MeV
B	2.33 MeV
C	37.6
D	- 196.9 MeV
E	51.3 MeV
m	285 MeV
$\tau$	$4.39 \text{ fm}^{-1}$
$\omega$	0.47
$K_0$	0.21 MeV

## RESULTS AND DISCUSSIONS

In the present work, the mass spectra of strange baryon resonances in hypercentral constituent quark model have been obtained. The calculated values of masses for few states are listed in table 2. Although our results of the resonances  $\Sigma(1940)$  and other excited states are in good agreement with the experimental results but while comparing with other theoretical models still some problems are found in the reproduction of the experimental masses of some baryons and in particular for the  $\Xi^*(1530)\text{P13}$  and the  $\Sigma(1660)\text{P11}$  resonances that's come out above the experimental values. The better agreement can be obtained by including some other factors dependence in the SU(6) breaking part. the overall good description of the spectrum which we obtain by the combination of potentials shows that our model can also be used to give a fair description of the energy of excited multiplet up to 2 GeV.

**Table2: Mass spectrum of strange baryons resonances**

Baryon	Status	State	M <sub>PDG</sub> [13]	M <sub>theor.</sub> [11]	M <sub>our calc.</sub>
$\Sigma$ (1193)P11	****	$^2 8_{1/2} [56, 0^+]$	1193	1193.0	1183.5
$\Sigma$ (1660)P11	***	$^2 8_{1/2} [56, 0^+]$	1630-1690	1703.7	1730.0
$\Sigma$ (1670)D13	****	$^2 8_{3/2} [70, 1^-]$	1665-1685	1798.7	1783.7
$\Sigma$ (1750)S11	***	$^2 8_{1/2} [70, 1^-]$	1730-1800	1798.7	1786.4
$\Sigma$ (1775)D15	****	$^4 8_{5/2} [70, 1^-]$	1770-1780	1913.6	1893.1
$\Sigma$ (1915)F15	****	$^2 8_{5/2} [56, 2^+]$	1900-1950	1906.4	1841.1
$\Sigma$ (1940)D13	***	$^4 8_{3/2} [70, 1^-]$	1900-1950	1913.6	1917.6
$\Sigma^*$ (1385)P13	****	$^4 10_{3/2} [56, 0^+]$	1383-1385	1371.6	1373.4
$\Sigma^*$ (2320)F17	****	$^4 10_{7/2} [56, 0^+]$	2025-2040	2085.0	2051.4
$\Xi$ (1318)P11	****	$^2 8_{1/2} [56, 0^+]$	1314-1316	1332.6	1340.0
$\Xi$ (1820)D13	***	$^2 8_{3/2} [70, 1^-]$	1818-1828	1938.3	1918.7
$\Xi^*$ (1530)P13	****	$^4 10_{3/2} [56, 0^+]$	1531-1532	1511.1	1553.0
$\Omega$ (1672)P03	****	$^4 10_{3/2} [56, 0^+]$	1672-1673	1650.7	1652.3

**REFERENCES**

1. B. Chakrabarti, A.bhattacharya, S.Mani, A. Sagari, *Acta Physica Polonica B*, Vol. 41, No. 1, 95-101(2010).
2. L. Ya. Glozman and D.O.Riska. *Phys. Rep.* C268, 263(1996).
3. N. Isgur and G. Karl, *Phys. Rev.* D18, 4187 (1978); D19, 2653(1979); D20, 1191 (1979).
4. R. Bijker, F. Iachello and A. Leviatan, *Ann. Phys.*(N.Y.) 236, 69 (1994)
5. M. M. Giannini, *Rep. Prog. Phys.* 54, 453 (1991).
6. M. M. Giannini, E. Santopinto, and A. Vassallo; arXiv:nucl-th/0506032v1, (2005).
7. M.Ferrars, M.M.Giannini,E.Santopinto, M. Pizzo, L.Tiator,and A.Vassallo; *Phys. Lett.* B364, 231(1995).
8. E.Santopinto, M.M.Giannini, F.Iachello, in *Symmetries in science VII*.ed.B. Gruber, Plenum Press, New York, p.231 (1995).
9. N.Salehi and A.A. Rajabi, *Mod. Phys. Lett.* A24, 2631(2009).
10. F.Gursey and L.A. Radicati, *Phys. Rev. Lett.*13,173(1964)
11. M.M. Giannini, E. Santopinto, and A.Vassallo; *Eur. Phys. J.* A25, 241-247 (2005).
12. R. Bijker, M.M. Giannini and E. Santopinto, *Eur.Phys.J.* A22,319 (2004).
13. Particle Data Group, *Journal of Physics* G37, 075021(2010).